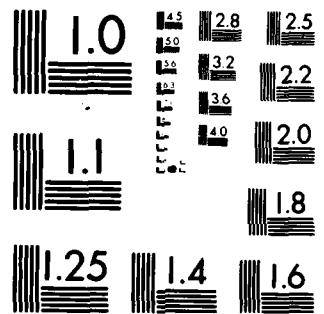


AD-A082 008 SACLANT ASW RESEARCH CENTRE LA SPEZIA (ITALY) F/6 8/3
A NUMERICAL SCHEME FOR PREDICTING THE LOCATION OF TIDALLY-GENERATED WAVE ENERGY
SEP 79 A J ELLIOTT

UNCLASSIFIED SACLANTCEN-SM-125

NL

END
DATE
FILED
4 80
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

ADA 082008

**SACLANT ASW
RESEARCH CENTRE
MEMORANDUM**

**A NUMERICAL SCHEME FOR PREDICTING THE LOCATION
OF TIDALLY-GENERATED FRONTS IN SHALLOW WATER**

by

ALAN J. ELLIOTT

DTIC
ELECTED
MAR 19 1980
S A D

1 SEPTEMBER 1979

DISTRIBUTION STATEMENT A

Approved for public release
Distribution Unlimited

LA SPEZIA, ITALY

NORTH
ATLANTIC
TREATY
ORGANIZATION

This document is unclassified. The information it contains is published subject to the conditions of the legend printed on the inside cover. Short quotations from it may be made in other publications if credit is given to the author(s). Except for working copies for research purposes or for use in official NATO publications, reproduction requires the authorization of the Director of SACLANTCEN.

JULG FILE 0128

80 3 14 044

This document is released to a NATO Government at the direction of the SACLANTCEN subject to the following conditions:

1. The recipient NATO Government agrees to use its best endeavours to ensure that the information herein disclosed, whether or not it bears a security classification, is not dealt with in any manner (a) contrary to the intent of the provisions of the Charter of the Centre, or (b) prejudicial to the rights of the owner thereof to obtain patent, copyright, or other like statutory protection therefor.
2. If the technical information was originally released to the Centre by a NATO Government subject to restrictions clearly marked on this document the recipient NATO Government agrees to use its best endeavours to abide by the terms of the restrictions so imposed by the releasing Government.

Published by



INITIAL DISTRIBUTIONMINISTRIES OF DEFENCE

	Copies
MOD Belgium	2
DND Canada	10
CHOD Denmark	8
MOD France	8
MOD Germany	15
MOD Greece	11
MOD Italy	10
MOD Netherlands	12
CHOD Norway	10
MOD Portugal	5
MOD Turkey	5
MOD U.K.	16
SECDEF U.S.	61

SOME FOREIGN AUTHORITIES

SECR Belgium	
SECR Canada	
SCMR Denmark	
SCMR Germany	
SCMR Greece	
SCMR Italy	
SCMR Netherlands	
SCMR Norway	
SCMR Portugal	
SCMR Turkey	
SCMR U.K.	
SCMR U.S.	
SECGEN Rep. SCMR	
NAMILCOM Rep. SCMR	
French Delegate SCMR	

NATO AUTHORITIES

Defence Planning Committee	3
NAMILCOM	2
SACLANT	10
SACLANTREPEUR	1
CINCMESTLANT/COMOCEANLANT	1
COMIBERLANT	1
CINCEASTLANT	1
COMSUBCLANT	1
COMMAIREASTLANT	1
SACEUR	2
CINCNORTH	1
CINCSOUTH	1
COMNAVSOUTH	1
COMSTRIKFORSOUTH	1
COMEDCENT	1
COMMARAIMED	1
CINCHAN	1

NATIONAL LIAISON OFFICERS

NLO Canada	
NLO Denmark	
NLO Germany	
NLO Italy	
NLO U.K.	
NLO U.S.	

NLR TO SACLANT

NLR Belgium	
NLR Canada	
NLR Germany	
NLR Greece	
NLR Italy	
NLR Norway	
NLR Portugal	
NLR Turkey	

Total initial distribution	233
SACLANTCEN Library	10
Stock	37
Total number of copies	280

(14)

SACLANTCEN [REDACTED] SM-125

NORTH ATLANTIC TREATY ORGANIZATION

SACLANT ASW Research Centre

Viale San Bartolomeo 400, I-19026 San Bartolomeo (SP), Italy.

tel: national 0187 503540
international + 39 187 503540

telex: 271148 SACENT I

(9)

Memorandum rept.,

(6)

A NUMERICAL SCHEME FOR PREDICTING THE LOCATION
OF TIDALLY-GENERATED FRONTS IN SHALLOW WATER.

by

(10)

Alan J. Elliott

(11)

1 Sept [REDACTED] 79

(12) 46

SEARCHED	INDEXED
SERIALIZED	FILED
UNCLASSIFIED	
JUL 1979	
FBI - WASHINGTON	
FEDERAL BUREAU OF INVESTIGATION	
U.S. DEPARTMENT OF JUSTICE	
1st / regular or special	
A	

This memorandum has been prepared within the SACLANTCEN Underwater Research Division, as part of Project 16.

C. Vettori

G.C. Vettori
Division Chief

312950

mtb

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	1
INTRODUCTION	1
1 THE HYDRODYNAMIC EQUATIONS	2
2 FINITE DIFFERENCES	7
3 APPLICATION TO THE SOUTHWESTERN APPROACHES TO THE ENGLISH CHANNEL	10
CONCLUSIONS	18
REFERENCES	19
APPENDIX A - THE PROGRAM LISTING	21

List of Figures

1. Grid scheme used in the model.	8
2. The coastal seas around the British Isles, showing the area covered by the model.	11
3. Details of the grid location points and the coastline configuration.	13
4. Bottom depths (metres) used in the model.	13
5. The observed amplitude and phase of the M_2 tidal component.	14
6. The computed amplitude and phase of the M_2 tide.	14
7. Calculated value of the stratification parameter $\log_{10}(u^3/h)$ for mean tidal conditions, the critical values are indicated.	15
8. Calculated stratification parameter for spring tides.	15
9. Calculated stratification parameter for neap tides.	16
10. Calculated stratification parameter due to an eastward wind of 16 m/s.	16
11. Calculated stratification parameter for weak tidal conditions.	17
12. Calculated stratification parameter for weak tides plus the wind used in Fig. 10.	17

A NUMERICAL SCHEME FOR PREDICTING THE LOCATION
OF TIDALLY-GENERATED FRONTS IN SHALLOW WATER

by

Alan J. Elliott

ABSTRACT

LOG₁₀ u TO THE 3RD POWER

A hydrodynamic numerical model is used to compute tidal currents and calculate the parameter $\log_{10}(u^3/h)$, where u is the maximum local tidal speed and h is the depth. A recent theory has shown that fronts, marking the boundary between stratified and isothermal water, can be expected to form where this parameter takes a critical value of about 2.2. Thus, if the amplitude and phase of the tidal elevations are known around a region of interest then the likely location of fronts can be predicted if the bottom topography is known. As an example of the method the scheme is applied to the Southwestern Approaches to the English Channel, and good agreement is obtained for the predicted location of a thermal front that has been observed at the western entrance during the summer months. In particular, the model shows that even eastward winds of Beaufort force 8, which occur for at least 10% of the time in this region, are unlikely to influence the location of the front. The computer program and instructions for its use are given in an appendix.

INTRODUCTION

During recent years there has been a growing body of evidence to suggest that thermal fronts can be generated in shallow water by the bottom-generated turbulence that is associated with tidal currents.

The first observations of this effect were made by Simpson [1] who observed a stratified region, persistent throughout the summer months, in the northwestern part of the Irish Sea. This region coincided with a zone characterized for its weak tidal currents, the remainder of the Irish Sea being noted for its strong tidal currents and vertically well-mixed water. In a further investigation [2] it was shown that the rate at which work is done by the tidal currents against the bottom friction is proportional to u^3 , where u is the tidal velocity, and that the criteria on whether the bottom-generated turbulence will be sufficiently strong to completely mix the water column vertically against buoyancy forces will depend on the ratio of u^3/h , where h is the local bottom depth. By using a numerical model to predict the tidal currents throughout the Irish Sea, and by comparing the contoured values of $\log_{10}(u^3/h)$ with the locations of known fronts, Simpson and Hunter showed that the critical value for the frontal location was around 2.2 (in cgs units).

This work was extended by Fearnhead [3] who used mean tidal charts to construct the contours of the stratification parameter in the waters surrounding the coast of the British Isles. Similar calculations of this kind have also been made by using tidal charts for conditions of mean spring tides and comparing the results with the frontal locations determined from satellite images [4, 5]. In a detailed numerical study of a front observed in the southern Irish Sea, James [6] showed the seasonal development of the frontal system and demonstrated that a front that forms during the neap part of the tidal cycle may not be destroyed during the following spring tide. More recently, Pingree and Griffiths [7] have used a high resolution numerical model to calculate the stratification parameter on the continental shelf around the British Isles, and good agreement was obtained between the predicted and observed frontal locations.

The purpose of this memorandum is to present a readily adapted scheme for making such predictions, and to give (in Appendix A) the listing of a computer program for calculating the stratification parameter $\log_{10}(u^3/h)$.

The advantage of using a hydrodynamic model to compute the tidal streams is that, although a rough estimate of the parameter u^3/h can be made by using tidal charts, this method cannot be used in regions where the tidal currents are not well known. However, in contrast to tidal currents, tidal elevations have been extensively studied for hundreds of years and the details, for almost all regions, can readily be found in the open literature. Taking advantage of this fact, we can use sea level information as input to the hydrodynamic model, i.e. specify the rise and fall of the tide around the boundary, and use the model to calculate the resulting interior tidal currents and the parameter u^3/h in regions where the tidal currents themselves may not be well known.

1 THE HYDRODYNAMIC EQUATIONS

The derivation of the depth-integrated form of the two-dimensional hydrodynamic equations can be found in numerous published papers and standard texts ([8, 9 & 10], for example). However, for completeness, a brief (non-rigorous) account of the derivation is given here. The basic equations are (with the usual notation):

$$\frac{du_1}{dt} + fu_2 = - \frac{1}{\rho} \frac{\partial p}{\partial x_1}$$

$$\frac{du_2}{dt} - fu_1 = - \frac{1}{\rho} \frac{\partial p}{\partial x_2}$$

$$\frac{\partial p}{\partial x_3} = \rho g$$

and

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0,$$

where right-handed axes have been chosen, and the z-axis is taken as positive downwards. If we partition each velocity component into a mean and fluctuating part so that

$$u_i = \bar{u}_i + u'_i ,$$

and then take the time average of the equations over an interval that is long compared with the time scale of the fluctuations, we obtain

$$\frac{\partial \bar{u}_1}{\partial t} + \frac{\partial}{\partial x_1} (\bar{u}_1 \bar{u}_1) + \frac{\partial}{\partial x_2} (\bar{u}_1 \bar{u}_2) + \frac{\partial}{\partial x_3} (\bar{u}_1 \bar{u}_3) + f \bar{u}_2 = - \frac{1}{\rho} \frac{\partial p}{\partial x_1}$$

and

$$\frac{\partial \bar{u}_2}{\partial t} + \frac{\partial}{\partial x_1} (\bar{u}_1 \bar{u}_2) + \frac{\partial}{\partial x_2} (\bar{u}_2 \bar{u}_2) + \frac{\partial}{\partial x_3} (\bar{u}_2 \bar{u}_3) - f \bar{u}_1 = - \frac{1}{\rho} \frac{\partial p}{\partial x_2}$$

(N.B. We have neglected the mean quantity non-linear terms). If we now drop the overbar, and use eddy coefficients to relate the turbulent stresses to the gradients of the mean quantities, then we obtain:

$$\frac{\partial u_1}{\partial t} - \frac{\partial}{\partial x_1} (N_1 \frac{\partial u_1}{\partial x_1}) - \frac{\partial}{\partial x_2} (N_2 \frac{\partial u_1}{\partial x_2}) - \frac{\partial}{\partial x_3} (N_3 \frac{\partial u_1}{\partial x_3}) + f u_2 = - \frac{1}{\rho} \frac{\partial p}{\partial x_1} \quad [\text{Eq. 1}]$$

and

$$\frac{\partial u_2}{\partial t} - \frac{\partial}{\partial x_1} (N_1 \frac{\partial u_2}{\partial x_1}) - \frac{\partial}{\partial x_2} (N_2 \frac{\partial u_2}{\partial x_2}) - \frac{\partial}{\partial x_3} (N_3 \frac{\partial u_2}{\partial x_3}) - f u_1 = - \frac{1}{\rho} \frac{\partial p}{\partial x_2} \quad [\text{Eq. 2}]$$

The hydrostatic equation, integrated from the surface ($x_3 = -\eta$) down to a depth x_3 becomes

$$p(x_3) - p_a = \int_{-\eta}^{x_3} \rho dx_3 ,$$

and therefore

$$\frac{\partial p}{\partial x_1} = g \frac{\partial}{\partial x_1} \left[\int_{-\eta}^{x_3} \rho dx_3 \right] = g \frac{\partial}{\partial x_1} [(x_3 + \eta) \bar{\rho}] ,$$

where

$$\bar{\rho} = \frac{1}{(x_3 + \eta)} \int_{-\eta}^{x_3} \rho dx_3 ,$$

(we have neglected the horizontal atmospheric pressure gradients). Therefore,

$$\begin{aligned}\frac{\partial p}{\partial x_1} &= g \left[\frac{\partial}{\partial x_1} (x_3 + \eta) \bar{\rho} + (x_3 + \eta) \frac{\partial \bar{\rho}}{\partial x_1} \right] \\ &= g \left[\frac{\partial \eta}{\partial x_1} \bar{\rho} + (x_3 + \eta) \frac{\partial \bar{\rho}}{\partial x_1} \right].\end{aligned}$$

Hence,

$$\frac{\partial p}{\partial x_1} = g \bar{\rho} \frac{\partial \eta}{\partial x_1} + g(x_3 + \eta) \frac{\partial \bar{\rho}}{\partial x_1},$$

which states that the horizontal pressure gradient has two parts: one part due to the slope of the free surface, and the other part due to the horizontal variations in density.

Hence, the pressure term on the right hand side of Eq. 1 becomes

$$-\frac{1}{\rho} \frac{\partial p}{\partial x_1} = -g \frac{\bar{\rho}}{\rho} \frac{\partial \eta}{\partial x_1} - \frac{g}{\rho} (x_3 + \eta) \frac{\partial \bar{\rho}}{\partial x_1}. \quad [\text{Eq. 3}]$$

The next stage is to integrate the equations vertically and express the variables in terms of depth-mean quantities. For simplicity it will be assumed that the water is well-mixed vertically. Under these circumstances $\rho = \bar{\rho}$ (at all depths) and therefore

$$\int_{-\eta}^d \left[-g \frac{\bar{\rho}}{\rho} \frac{\partial \eta}{\partial x_1} \right] dx_3 = -g \frac{\partial \eta}{\partial x_1} (d + \eta)$$

and

$$-\frac{g}{\rho} \int_{-\eta}^d \left[(x_3 + \eta) \frac{\partial \bar{\rho}}{\partial x_1} \right] dx_3 = -\frac{g}{\rho} \frac{(d + \eta)^2}{2} \frac{\partial \bar{\rho}}{\partial x_1}.$$

Consequently, the vertical integration of the pressure terms of Eq. 3 gives

$$-g(d + \eta) \frac{\partial \eta}{\partial x_1} - \frac{g}{\rho} \frac{(d + \eta)^2}{2} \frac{\partial \bar{\rho}}{\partial x_1},$$

with similar terms in the x_2 momentum equation. To integrate the velocity terms over the depth requires the use of kinematic boundary conditions at the top and bottom boundaries.

If $\phi(x_1, x_2, x_3) = 0$ is the equation of the surface and $\underline{u} = (u_1, u_2, u_3)$ is the velocity, then the surface condition is

$$\underline{u} \cdot \nabla \phi = -\frac{\partial \eta}{\partial t}. \quad [\text{Eq. 4}]$$

Since the surface is given by

$$x_3 = -\eta(x_1, x_2),$$

then

$$\phi(x_1, x_2, x_3) = \eta(x_1, x_2) - x_3$$

and

$$\nabla \phi = \left(\frac{\partial \eta}{\partial x_1}, \frac{\partial \eta}{\partial x_2}, -1 \right).$$

Therefore, Eq. 4 gives

$$\frac{\partial \eta}{\partial t} + u_1 \frac{\partial \eta}{\partial x_1} + u_2 \frac{\partial \eta}{\partial x_2} + u_3 = 0 \quad \text{at } x_3 = -\eta. \quad [\text{Eq. 5}]$$

At the bottom,

$$x_3 = d(x_1, x_2)$$

and

$$\phi(x_1, x_2, x_3) = d(x_1, x_2) - x_3.$$

Therefore $\underline{u} \cdot \nabla \phi = 0$ (i.e. no flow through the bottom) gives that

$$u_1 \frac{\partial d}{\partial x_1} + u_2 \frac{\partial d}{\partial x_2} - u_3 = 0 \quad \text{at } x_3 = d. \quad [\text{Eq. 6}]$$

If the velocity terms in Eqs. 1 & 2 are now integrated over the depth, and the kinematic conditions of Eqs. 5 & 6 are used, then the momentum equations become (in terms of depth-mean quantities):

$$\begin{aligned} \frac{\partial}{\partial t} [(d+\eta)u_1] &= \frac{\partial}{\partial x_1} [(d+\eta) N \frac{\partial u_1}{\partial x_1}] + \frac{\partial}{\partial x_2} [(d+\eta) N \frac{\partial u_1}{\partial x_2}] \\ &\quad + F_{x_S} - F_{x_B} - f(d+\eta)u_2 \\ &\quad - g(d+\eta) \frac{\partial \eta}{\partial x_1} - \frac{g}{\rho} \frac{(d+\eta)^2}{2} \frac{\partial p}{\partial x_1} \end{aligned} \quad [\text{Eq. 7}]$$

(F_{x_S} and F_{x_B} are the components of the surface and bottom stresses) and

$$\begin{aligned} \frac{\partial}{\partial t} [(d+\eta)u_2] &= \frac{\partial}{\partial x_1} [(d+\eta) N \frac{\partial u_2}{\partial x_1}] + \frac{\partial}{\partial x_2} [(d+\eta) N \frac{\partial u_2}{\partial x_2}] \\ &+ F_{y_S} - F_{y_B} + f(d+\eta)u_1 \\ &- g(d+\eta) \frac{\partial \eta}{\partial x_2} - \frac{g}{\rho} \frac{(d+\eta)^2}{2} \frac{\partial p}{\partial x_2}. \end{aligned} \quad [\text{Eq. 8}]$$

The final equation that is required is the depth-integrated form of the continuity equation, which will be used to compute the surface elevation. If v is an arbitrary volume of fluid, fixed in space, of density ρ and surface area A , then

$$\frac{d}{dt} \int_v \rho dv = - \int_A \rho u \cdot ds + \int_v \delta dv,$$

where δ is the source of fluid/unit volume/unit time.

This gives

$$\int_v \left[\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \underline{u}) - \delta \right] dv = 0,$$

and hence

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \underline{u}) = \delta.$$

If the flow is assumed to be incompressible, this reduces to

$$\operatorname{div}(\underline{u}) = \frac{\delta}{\rho},$$

i.e.

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{\delta}{\rho}. \quad [\text{Eq. 9}]$$

If Eq. 9 is now integrated vertically over the water column and use is made of the kinematic conditions of Eqs. 5 & 6, we obtain

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x_1} [(d+\eta)u_1] + \frac{\partial}{\partial x_2} [(d+\eta)u_2] = (d+\eta) \frac{\delta}{\rho}. \quad [\text{Eq. 10}]$$

This equation, along with the momentum equations 7 and 8, are the ones solved by the numerical scheme. In the present analysis the horizontal density gradient terms in Eqs. 7 & 8 were neglected.

2 FINITE DIFFERENCES

Equations 7, 8, & 10 were solved using centred differences on a regular grid (Fig. 1). (Note that the grid uses left-handed axes; the only effect of this is to change the sign of the coriolis terms in Eqs. 7 & 8). Linear interpolation was used to derive the values of variables required at points other than grid points. A leap-frog scheme was used for the time integration, which involves the storage of three time levels, the time derivatives were evaluated using levels one and three, while the right-hand side of Eqs. 7, 8 & 10 were evaluated at the middle time level. The only exceptions were the diffusive (friction) terms; these were lagged back one time step for reasons of stability.

As an example, the finite difference form of the continuity equation will be derived (the finite difference form of the momentum equations are given in Appendix A). Let

$$\frac{\partial \eta}{\partial t} = (d+\eta) \frac{\delta}{\rho} - \frac{\partial}{\partial x_1} [(d+\eta) u_1] - \frac{\partial}{\partial x_2} [(d+\eta) u_2]$$

be written as

$$\frac{\partial \eta}{\partial t} = c_1 + c_2 + c_3,$$

then

$$c_1 = (d+\eta) \frac{\delta}{\rho}$$

where δ is the source/unit volume/unit time.

If the quantity δ' is the total source/grid box/unit time then

$$\delta' = (d+\eta) (2\Delta x) (2\Delta y) \delta.$$

Therefore

$$c_1 = \frac{(d+\eta)\delta'}{\rho(d+\eta) 4\Delta x \Delta y} = \frac{\delta'}{4\rho \Delta x \Delta y}.$$

Consequently, the finite difference form of this is simply

$$c_1 = \delta'(i,j) / [(4\Delta x \Delta y) \rho(i,j)], \quad TL = 2$$

where $\rho(i,j)$ denotes the value of ρ at the grid coordinate (i,j) and $TL = 2$ signifies time level 2.

$$c_2 = - \frac{\partial}{\partial x_1} [(d+\eta) u_1]$$

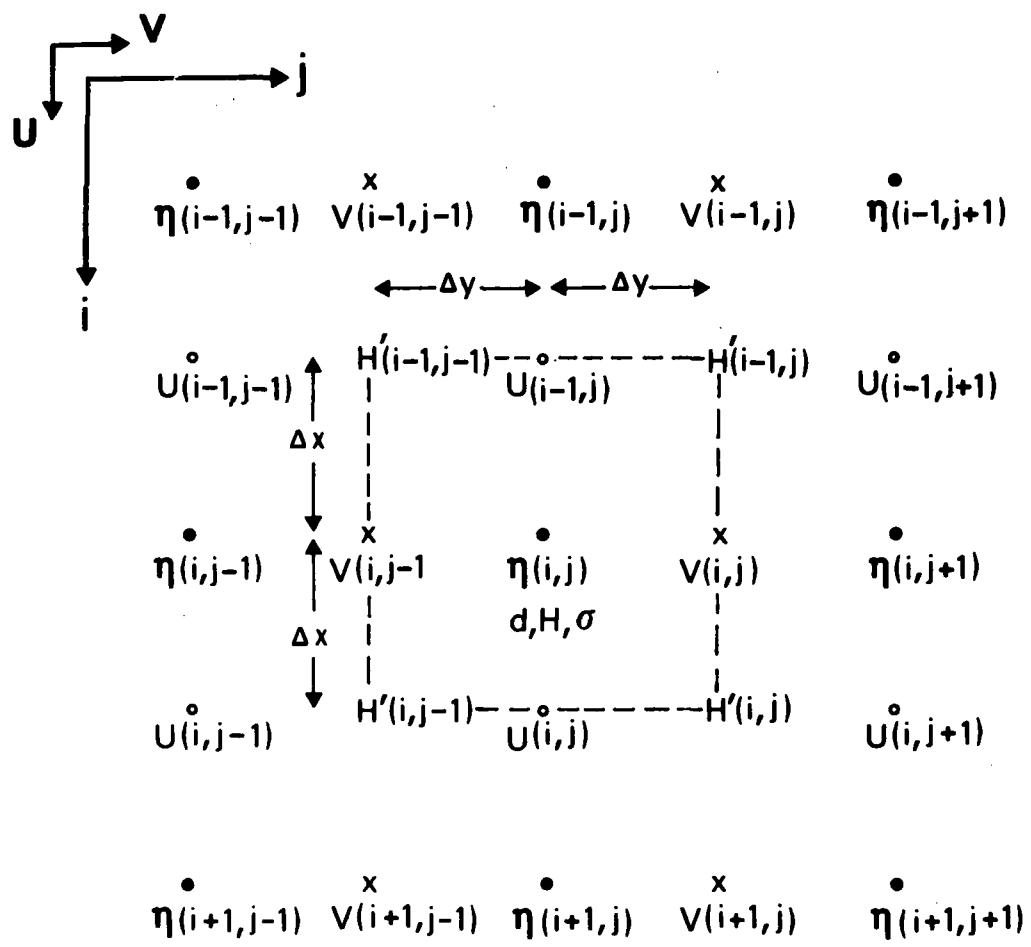


FIG. 1 GRID SCHEME USED IN THE MODEL

Let $(d+\eta) = H$, the total depth, then

$$c_2 = -\frac{\partial}{\partial x_1} [H u_1] \\ = \frac{-[(H(i,j) + H(i+1,j)) v(i,j) - (H(i-1,j) + H(i,j)) v(i-1,j)]}{2} \\ \frac{2\Delta x}{2}$$

Therefore

$$c_2 = \frac{-[(H(i,j) + H(i+1,j)) v(i,j) - (H(i-1,j) + H(i,j)) v(i-1,j)]}{(4\Delta x)}, \quad TL = 2.$$

Similarly,

$$c_3 = \frac{[(H(i,j) + H(i,j+1)) v(i,j) - (H(i,j-1) + H(i,j)) v(i,j-1)]}{(4\Delta y)}, \quad TL = 2.$$

Consequently,

$$\frac{\partial \eta}{\partial t} = c_1 + c_2 + c_3$$

becomes

$$\frac{n(i,j)|_{TL=3} - n(i,j)|_{TL=1}}{2\Delta t} = (c_1 + c_2 + c_3)|_{TL=2}.$$

Therefore,

$$n(i,j)|_{TL=3} = n(i,j)|_{TL=1} + 2\Delta t(c_1 + c_2 + c_3)|_{TL=2}.$$

This is the equation used to update the surface elevations (see subroutine ETA3 in Appendix A); comparable expressions can be derived in order to update the U and V velocity components (see UVEL and VVEL in Appendix A). In certain circumstances (e.g. when a point lies on or near an open boundary) the full form of the finite difference equations are not used because certain terms are neglected. The points were coded as follows:

For elevation,

- (1) outside the computational grid, do not update
(i.e. the point lies inland).
- (2) a boundary value, specify through a boundary condition.
- (3) computational point.

For velocity,

- (1) outside the grid or on a solid boundary, do not update.
- (2) near an open north-south boundary, neglect the term involving $\frac{\partial}{\partial x_2}$ and coriolis.
- (3) near an open east-west boundary, neglect the term in $\frac{\partial}{\partial x_1}$ and coriolis.
- (4) both of the above, i.e. near an open corner of the grid.
- (5) normal interior point, update using the full form of the equations.

The finite difference form of the equations were tested by applying the model to simple problems having known solutions. For example, the seiche motion of a rectangular lake, the advance of a wave along a rectangular canal (with and without rotation), the wind-induced set-up of a rectangular bay, and similar problems. In all cases, the difference between the known and computed solutions did not exceed a few percent of the analytical solution. The application of the model to the interpretation of some field observations has been given in a previous memorandum [11], showing a good agreement between the observed and predicted response.

In the present study the amplitude and phase of the surface tide is prescribed around the open boundary of the model. This is a sufficient boundary condition to drive the tidal oscillations (i.e. tidal currents and elevations) in the interior of the model. After allowing at least five tidal cycles for the system to approach a quasi-steady state, the subroutine PARAM (see Appendix A) computes the value of $\log_{10}(u^3/h)$ at each elevation point by averaging the surrounding velocities. At each time step, and for each grid point, the present value of the parameter is compared with the previous maximum and the value is updated if a new maximum has been reached. In this way, the maximum value of $\log_{10}(u^3/h)$ is calculated at every computational elevation point.

3 APPLICATION TO THE SOUTHWESTERN APPROACHES TO THE ENGLISH CHANNEL

As an example of the scheme, this chapter outlines the steps involved in applying the model and calculating the stratification parameter in the Southwestern Approaches. The area covered by the model is shown by the insert in Fig. 2; it extends from the Straits of Dover in the east and the north Channel of the Irish Sea in the north to the large open boundary south of Ireland in the southwest. The eastern and northern boundaries were placed at narrow sections to allow a better definition of the boundary conditions. Open boundaries, like the one to the southwest, should be avoided whenever possible because of the uncertainty involved in specifying the tidal constants in deep water. In the present circumstances, however, this choice of the boundary was unavoidable.

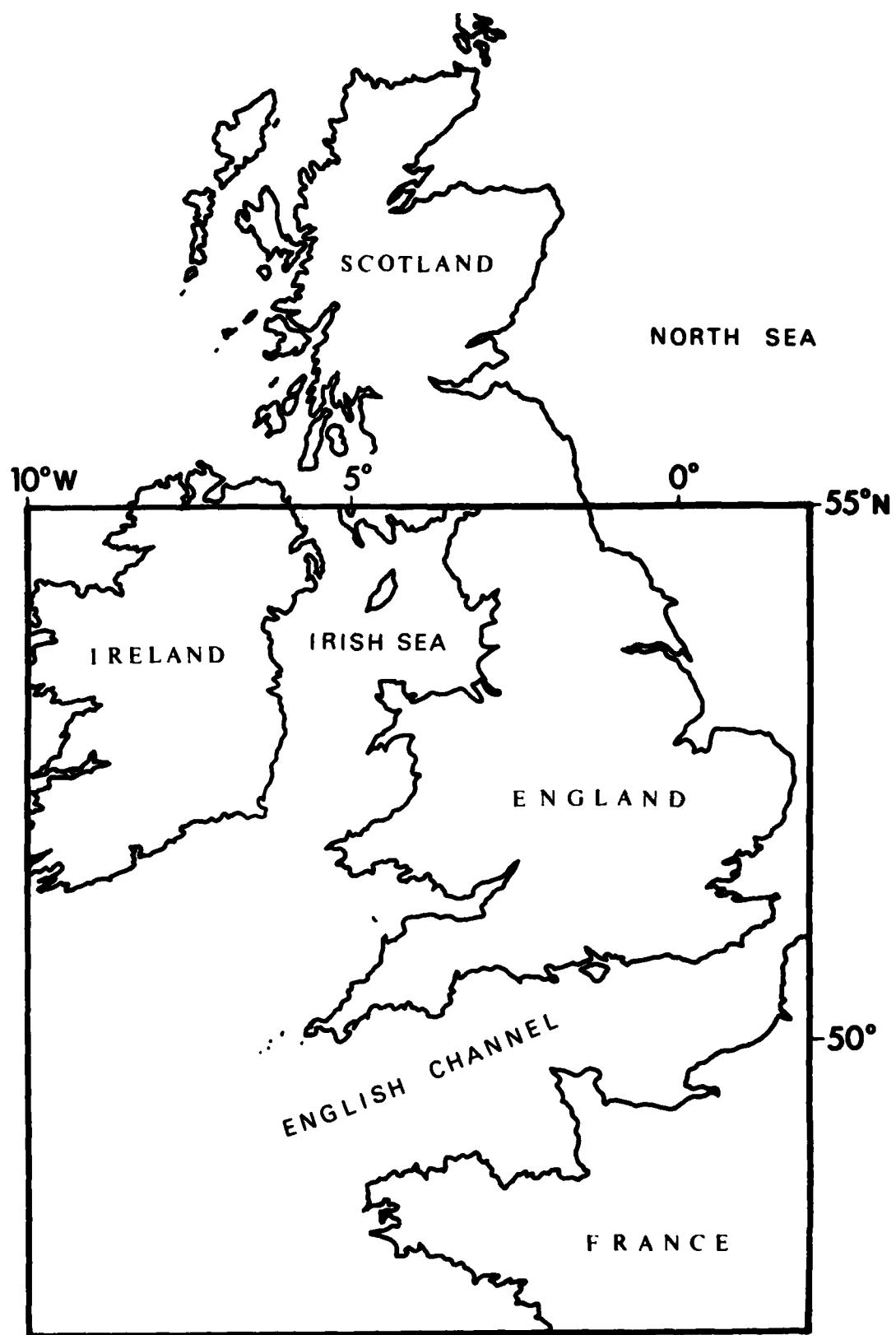


FIG. 2 THE COASTAL SEAS AROUND THE BRITISH ISLES, SHOWING
THE AREA COVERED BY THE MODEL

The grid points of the model and the solid boundaries (i.e. the coastline) are shown in Fig. 3; a very crude grid has been used for the purposes of demonstration, to obtain more accurate results it would be necessary to use a much smaller grid spacing (see, for example, [7]). The corresponding water depths, defined at the elevation points, are shown in Fig. 4. The amplitudes and phases of the M_2 tide around the

open boundary were taken from Flather [12] and are shown in Fig. 5 for the whole of the model region. (The boundary values are given in the subroutine BC). The major uncertainty (apart from the effect of the crude coastline representation) was the extrapolation of the tidal boundary conditions along the large open boundary. This was done by a process of trial and error until a reasonable agreement between the observed and computed tidal behaviour was obtained for the western entrance to the Channel. The computed phase and amplitude distributions are shown in Fig. 6. A fairly good agreement was achieved in the Approaches to the Channel and in the Channel itself, and also within the Bristol Channel. Note, however, that the agreement between the observed and computed distributions was poor throughout most of the Irish Sea.

The amplitudes shown in Figs. 5 and 6 are for the M_2 component alone. As part of the present study it was decided to investigate the variation of the stratification parameter (and hence the predicted frontal location) during the spring/neap cycle of the tide. The proper way to do this would be to specify the amplitude of both M_2 and S_2 around the open boundary, to run the model to simulate about one month of elapsed time, and to monitor the motion of the frontal location during the spring/neap cycle. A simpler way is to run the model for three cases:

- a. Mean tidal conditions: amplitudes around the boundary given by M_2 alone.
- b. Spring tides: amplitudes around the boundary given by $(M_2 + S_2)$.
- c. Neap tides: amplitudes around the boundary given by $(M_2 - S_2)$.

Typical ratios of the $M_2:S_2$ amplitudes for the region were obtained from Heaps [13], and the open boundary amplitudes were scaled, to reproduce the three cases given above, in the ratios 1.0 : 1.4 : 0.6.

The results are shown in Figs. 7, 8 and 9, respectively. They suggest that the locations in which the front might possibly form cover a considerable proportion of the area at the western entrance to the Channel. However, as pointed out by James [6], the persistancy of a front during the spring/neap cycle is not well established. It is encouraging, however, that the calculated mean location of the front (Fig. 7) is in general agreement with the location determined from observations [5,7,13].

Some additional tests were made to determine the likely effect of storm-generated currents and their contribution to the stratification parameter. The strongest winds in this region are directed towards the east [14]; winds of Beaufort force 8 or greater occurring on average 35-40 days/year.

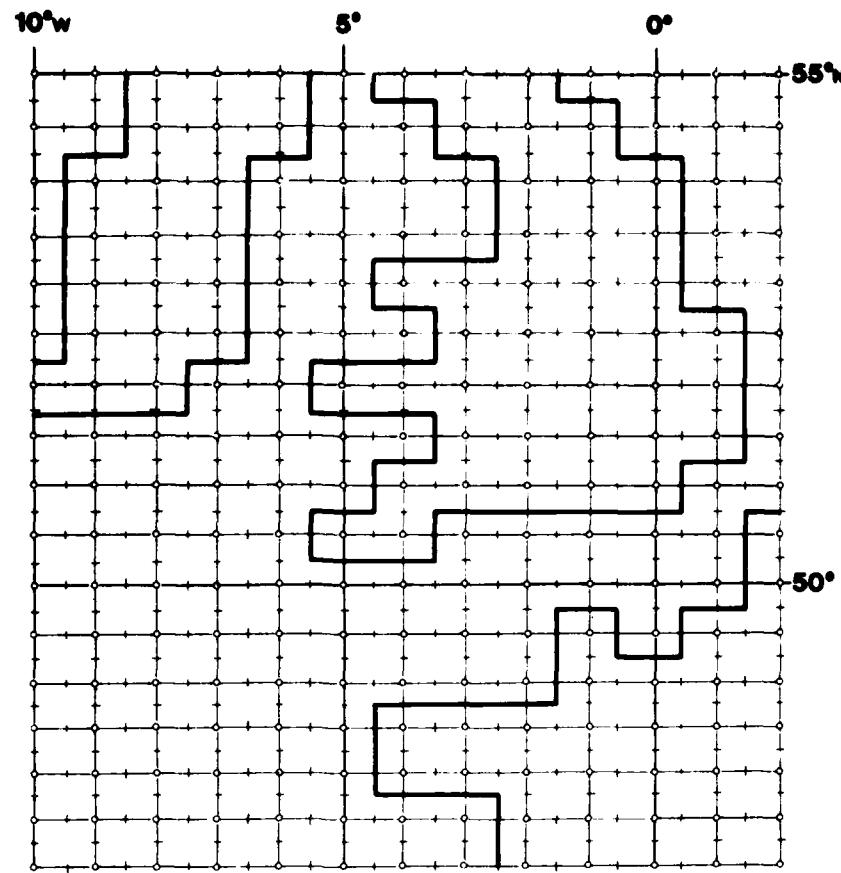


FIG. 3 DETAILS OF THE GRID LOCATION POINTS AND THE COASTLINE CONFIGURATION

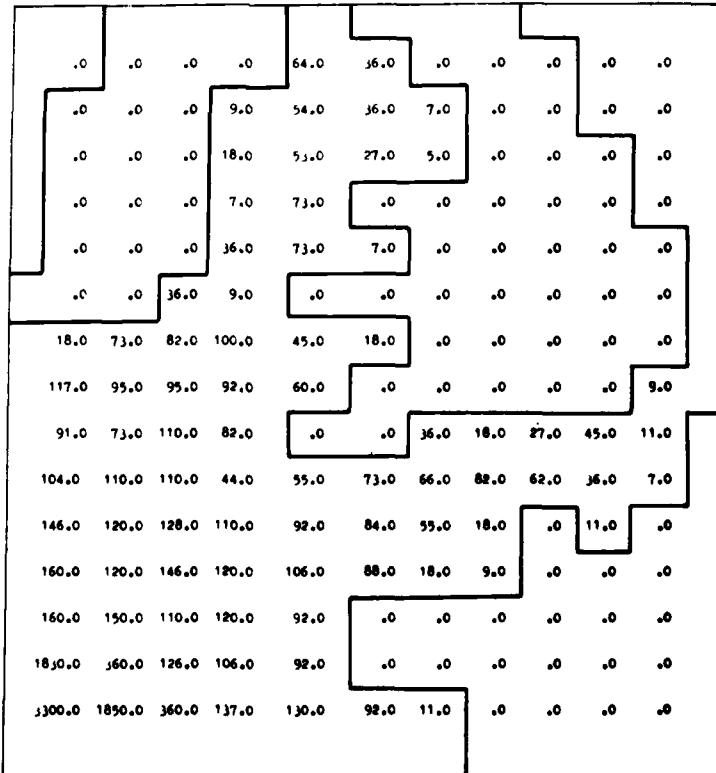


FIG. 4 BOTTOM DEPTHS (metres) USED IN THE MODEL

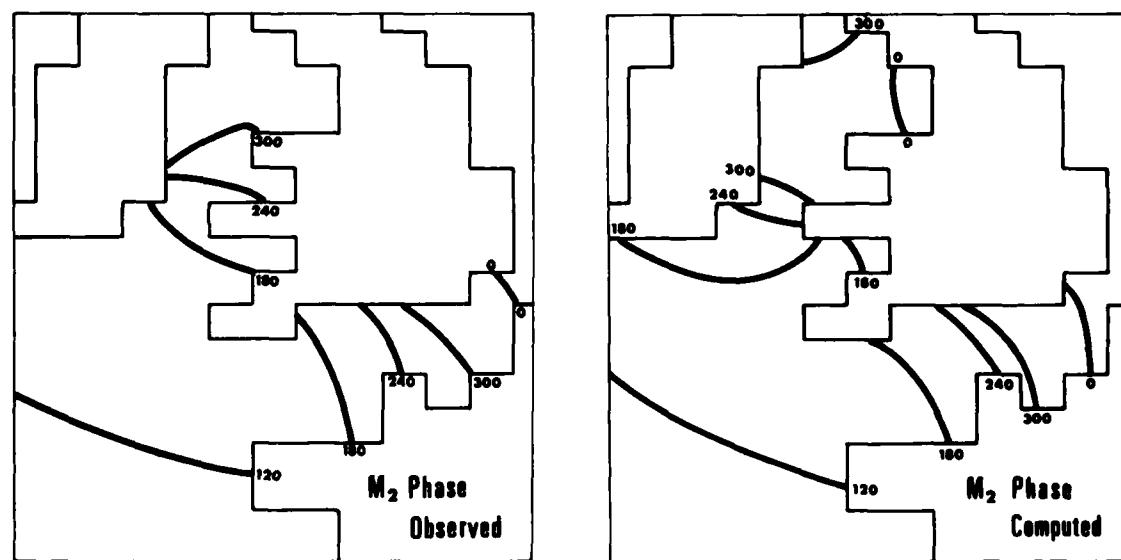
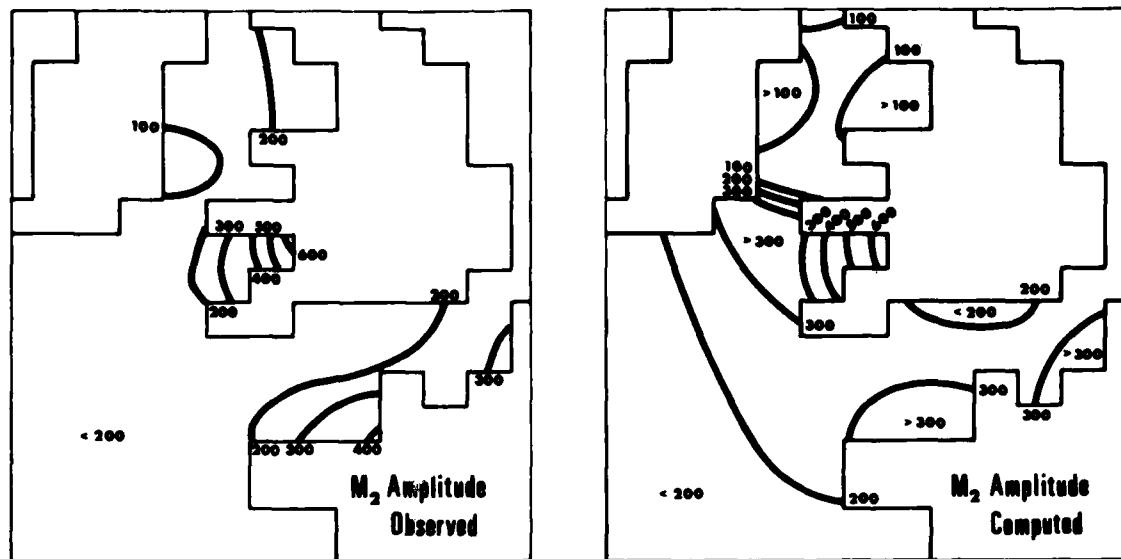


FIG. 5
THE OBSERVED AMPLITUDE AND PHASE OF
THE M_2 TIDAL COMPONENT
(taken from [12])

FIG. 6
THE COMPUTED AMPLITUDE AND PHASE OF
THE M_2 TIDE

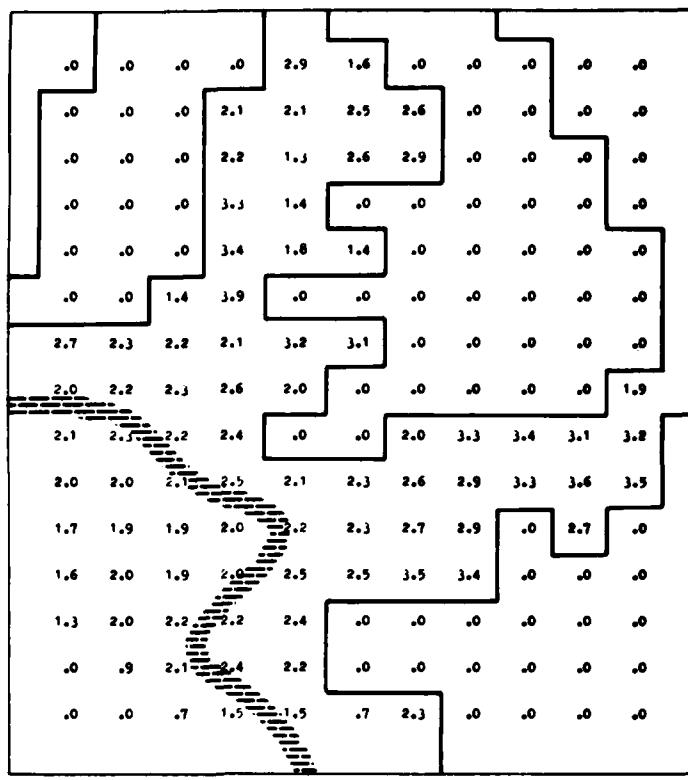


FIG. 7 CALCULATED VALUE OF THE STRATIFICATION PARAMETER $\log_{10}(u^3/h)$ FOR MEAN TIDAL CONDITIONS, THE CRITICAL VALUES ARE INDICATED

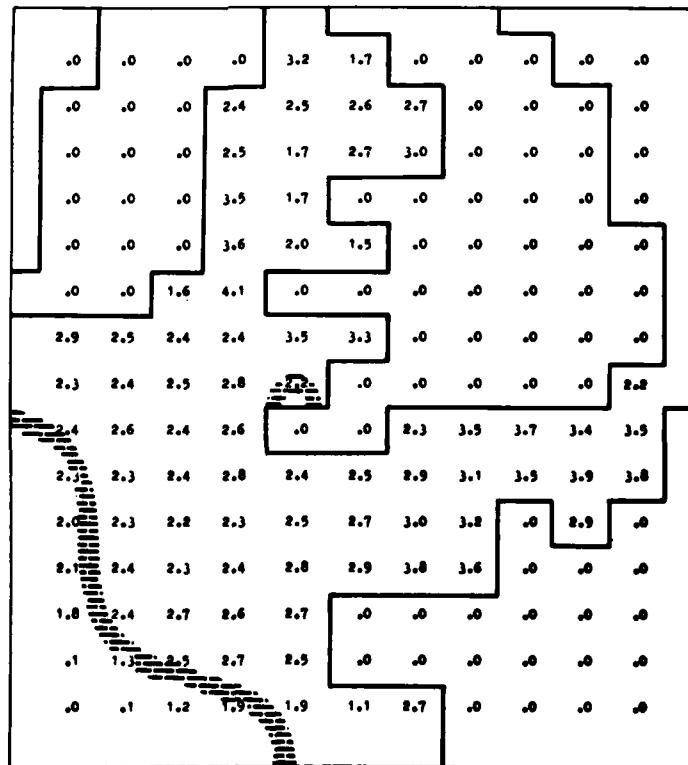


FIG. 8 CALCULATED STRATIFICATION PARAMETER FOR SPRING TIDES

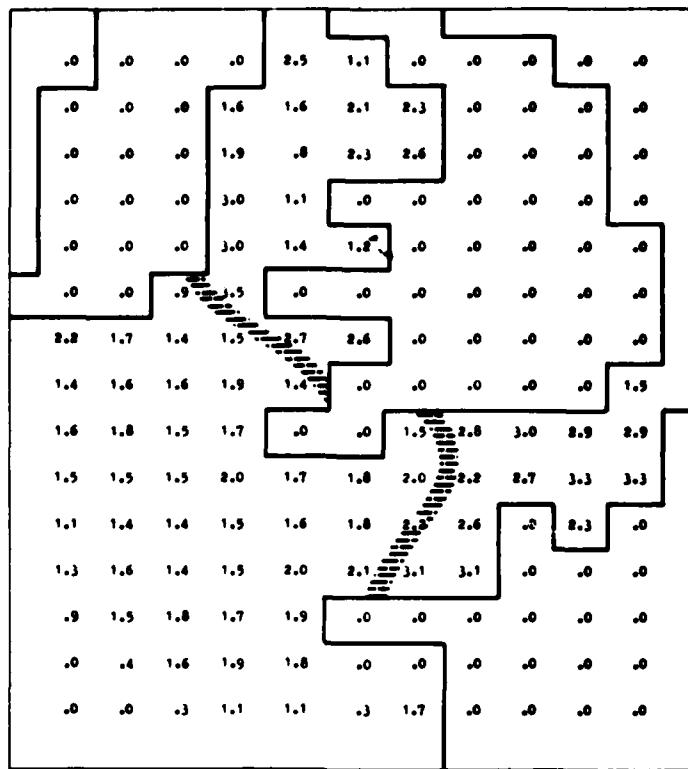


FIG. 9 CALCULATED STRATIFICATION PARAMETER FOR NEAP TIDES

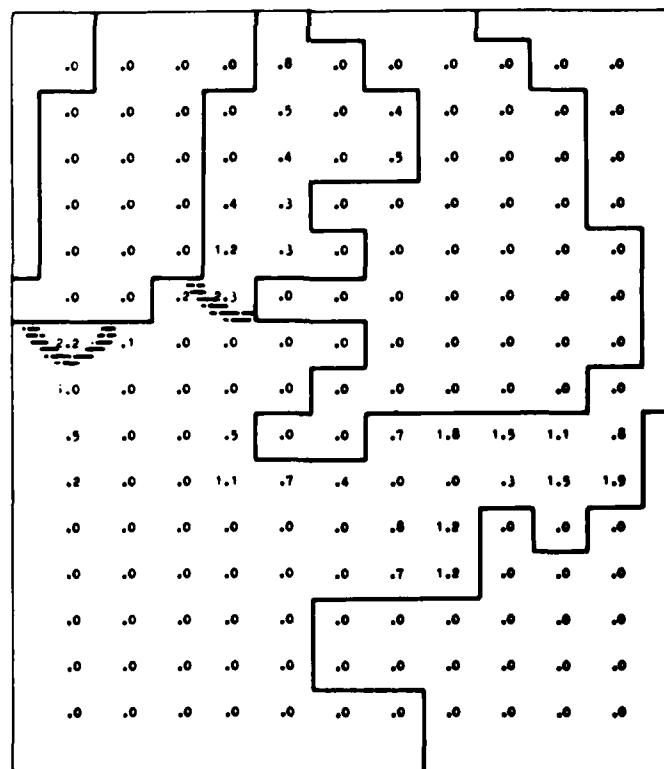


FIG. 10 CALCULATED STRATIFICATION PARAMETER DUE TO AN EASTWARD WIND OF 16 m/s

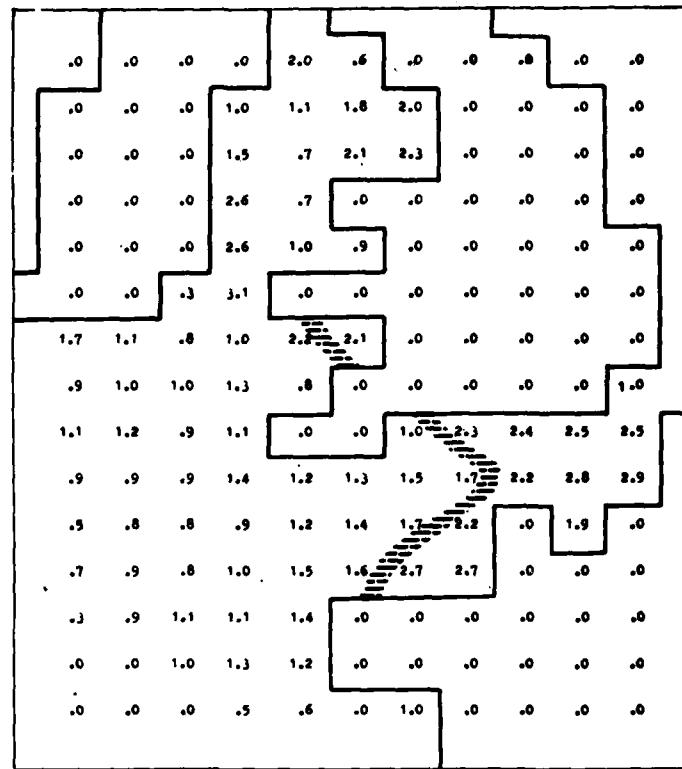


FIG. 11 CALCULATED STRATIFICATION PARAMETER FOR WEAK TIDAL CONDITIONS
(boundary amplitudes equal to 0.3 mean tidal amplitudes)

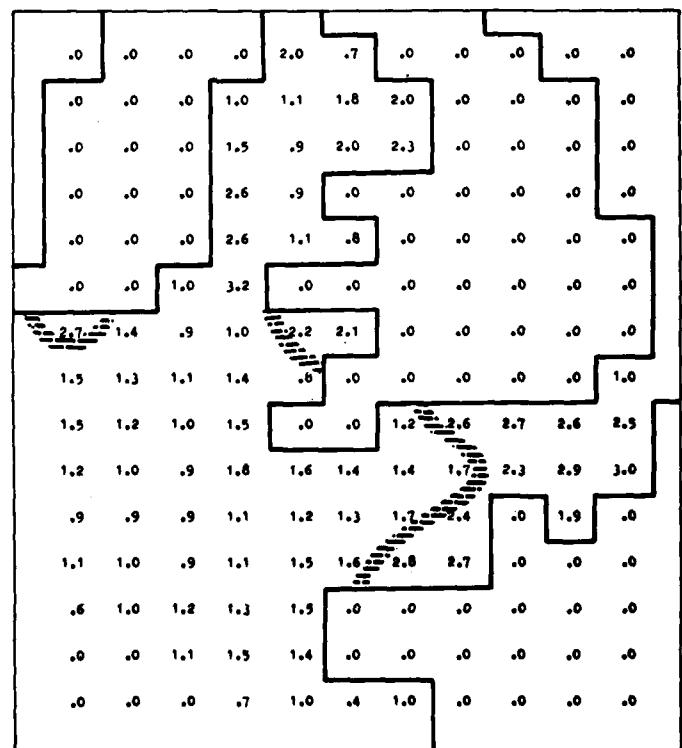


FIG. 12 CALCULATED STRATIFICATION PARAMETER FOR WEAK TIDES PLUS
THE WIND USED IN FIG. 10.

This corresponds to an eastward wind of around 16 m/s, equivalent to a surface stress of 4 dyn/cm². To simulate the effect of such storms the model was run with a constant eastward wind stress and the wind-driven currents computed. As in the previous calculations the stratification parameter $\log_{10}(u^3/h)$ was estimated, based this time on the storm driven flow. (Note that this calculation ignores the downward vertical mixing due to the surface stirring of the wind; the model only takes account of the bottom-generated turbulence). The wind-driven flow was, almost everywhere, too weak to raise the stratification parameter to the critical value (Fig. 10). The only two exceptions were a location off the south coast of Ireland (where there was an easterly flow at about 1 kn (51 cm/s) in the shallow water) and in the Channel at the southern entrance to the Irish Sea (where the flow was southward at about 1 kn). Throughout the whole of the English Channel the wind-driven currents alone were too weak to cause the generation of a front. In general, the effect of wind driving was insignificant in comparison with the tidal effects, even weak tides being sufficient to mask the influence of the wind. To illustrate this point, Fig. 11 shows the effect of weak tides (boundary amplitudes set to 0.3 of the mean amplitudes), while Fig. 12 shows the effect of weak tides plus wind on the stratification parameter. It is clear from the figures that the tides dominate over the wind.

CONCLUSIONS

The purpose of this memorandum has been to present a readily adaptable scheme for predicting the likely locations at which thermal fronts might be formed by the bottom turbulence associated with the tides. The advantage of the method is that it requires only a knowledge of the behaviour of the tidal elevations around the open boundary, the internal dynamics within the region of interest are then computed numerically. By way of an example, the method was applied to an area that includes the South-western Approaches to the English Channel. To establish the computational grid, digitize the depth, and code the grid points and coastline required about five working days, the calculations described in the body of this memorandum required about 12 minutes of computer time each on a Univac 1106. Consequently, to apply the scheme to a new region and make all of the necessary computations would require about 10 man-days of effort (this figure should be maybe doubled or trebled if the individual is not familiar with the model).

In the present study the model was adjusted only for the area near the western entrance to the English Channel. Better results would probably be obtained if a higher resolution grid was used, and if more effort was spent on calibrating the predicted tide. In its application to the Southwestern Approaches the greatest uncertainty probably arises during the extrapolation of the tidal amplitude and phase along the open boundary to the southwest.

REFERENCES

1. SIMPSON, J.H. Density stratification and microstructure in the western Irish Sea. *Deep Sea Research* 18, 1971: 309-319.
2. SIMPSON, J.H. and HUNTER, J.R. Fronts in the Irish Sea. *Nature* 250, 1974: 404-406.
3. FEARNHEAD, P.G. On the formation of fronts by tidal mixing around the British Isles. *Deep Sea Research* 22, 1975: 311-321.
4. SIMPSON, J.H. and PINGREE, R.D. Shallow sea fronts produced by tidal stirring. In: BOWMAN, M.J. and ESAIAS, W.E., eds. *Oceanic Fronts in Coastal Processes*. New York, N.Y., Springer-Verlag, 1978: pp. 29-42.
5. SIMPSON, J.H., HUGHES, D.G., and MORRIS, N.C.G. The relation of seasonal stratification to tidal mixing on the continental shelf. In: ANGEL, M. ed. *A Voyage of Discovery*. Oxford, Pergamon, 1977: pp. 327-340.
6. JAMES, J.D. A model of the annual cycle of temperature in a frontal region of the Celtic Sea. *Estuarine and Coastal Marine Science* 5, 1977: 339-353.
7. PINGREE, R.D. and GRIFFITHS, D.K. Tidal fronts on the shelf seas around the British Isles. *J. Geophysical Research* 83, 1978: 4615-4622.
8. TRACOR Inc. Estuarine modelling: an assessment, Water Pollution Control Research Series 16070 DZV 02/71. Washington D.C. US Govt Printing Office, 1971. [Stock № 5501-0129]
9. HEAPS, N.S. A two-dimensional numerical sea model. *Philosophical Transactions Royal Society A* 265, 1969: 93-137.
10. HEAPS, N.S. Linearized vertically-integrated equations for residual circulation in coastal seas. *Deutsche Hydrographische Zeitschrift* 31, 1978: 147-169.
11. ELLIOTT, A.J. Response of the coastal waters of northwest Italy, SACLANTCEN SM-117. La Spezia, SACLANT ASW Research Centre, 1978. [AD A0 67254]
12. FLATHER, R.A. A tidal model of the north-west European continental shelf. *Memoire Société Royale Sciences de Liège* 6ème Ser, X; 1976: 141-164.
13. HEAPS, N.S. and JONES, J.E. Recent storm surges in the Irish sea. In: NIHOUL, J.C.J. ed. *Marine Forecasting*. Amsterdam, Neth., Elsevier, in press.
14. SIMPSON, J.H., ALLEN, C.M., and MORRIS, N.C.G. Fronts on the continental shelf. *J. Geophysical Research* 83, 1978: 4607-4614.
15. COUPER, A.D., DRAPER, L. and WALKER, J.M. The Celtic Sea: meteorological and oceanographic conditions. Cardiff, U.K., Dept of Maritime Studies, University College, 1974.

APPENDIX A
THE PROGRAM LISTING

This appendix contains a listing of the FORTRAN program and subroutines required to make the calculations described in the main text. Comment cards have been inserted in the body of the program at the points where each input variable is introduced.

```

1      C
2      C      MAIN PROGRAM.
3      C
4      C      LOCATION OF TIDALLY GENERATED FRONTS.
5      C
6      C
7      C      WRITTEN BY:
8      C
9      C      A. J. ELLIOTT
10     C      SACLANTCEN, LA SPEZIA.
11     C      SEPTEMBER 1978.
12     C
13     C      THIS VERSION USES THE CALCOMP 960 PLOTTER.
14     C
15     C      C..... THE FOLLOWING COMMON BLOCKS SHOULD BE
16     C      C..... INSERTED AT THE BEGINNING OF EVERY SUBROUTINE.
17     C
18     C      COMMON/C1/ U(20,30,3),V(20,30,3),S(20,30,3),C(20,30,3)
19     C      COMMON/C2/ ETA(20,30,3),D(20,30),H(20,30,3),SIG(20,30)-
20     C      $SIG(20,30)
21     C      COMMON/C3/ UBAR(20,30),VBAR(20,30),SBAR(20,30)-
22     C      $CBAR(20,30),EBAR(20,30)
23     C      COMMON/C4/ IU(20,30),IV(20,30),IE(20,30),IS(20,30)
24     C      COMMON/C5/ K1(20,30),K2(20,30),N1,N2
25     C      COMMON/C6/ G,DX,DY,DT,TOR,WX,WY,NX,NY,NX1,NY1,
26     C      $NN,TC,TM,NTC,F
27     C      COMMON/C7/ IBUF(10),XI(30),Y(30),Z(30+30),IPLOT1
28     C      $,7 LE V(50),LABC(10),LWGT(10)
29     C      COMMON/C8/ ROW(20,30,3)
30     C      COMMON/C9/ NSUM
31     C      COMMON/C10/ XA(20),YA(20),XB(20),YB(20),XC(20),YC(20),
32     C      $XD(20),YD(20),XE(20),YE(20),XF(20),YF(20)
33     C      REAL K1,K2,N1,N2
34     C
35     C      C.....
36     C
37     C
38     C
39     C      READ IN BASIC PARAMETERS.
40     C
41     C
42     C      DELTAX,DELTAY IN KM.
43     C
44     C      DELTAT IN SEC.
45     C
46     C      100 FORMAT(F10.0)
47     C      READ(5,100) DX
48     C      READ(5,100) DY
49     C      READ(5,100) DT
50     C
51     C      CONVERT TO CGS UNITS.
52     C
53     C      DX=DX*10.**5.
54     C      DY=DY*10.**5.
55     C
56     C      COMPUTATIONAL CONSTANTS.

```

```

57      C
58      C      BOTTOM FRICTION.
59      C
60      C      TOR=2.5*10.**(-3.)
61      C
62      C      G=981.
63      C      OMEGA=7.272*10.**(-5.)
64      C
65      C      RRLAT IS THE LATITUDE.
66      C
67      C      RRLAT=50.
68      C      RLAT=RRLAT/180.*3.14159
69      C      F=2.*OMEGA*SIN(RLAT)
70      C
71      C      NTC=INT(12.4224*3600./DT+0.0001)
72      C
73      C      NX AND NY ARE THE DIMENSIONS OF THE WORKING GRID.
74      C
75      101   FORMAT(I4)
76      C
77      C      READ(5,101) NX
78      C      READ(5,101) NY
79      C
80      C      NX 1=NX-1
81      C      NY 1=NY-1
82      C
83      C      NSTEP IS THE NO. OF ITERATIONS.
84      C
85      102   FORMAT(I6)
86      C      READ(5,102) NSTEP
87      C
88      C      NANS IS THE OUTPUT FREQUENCY.
89      C
90      C      READ(5,101) NANS
91      C
92      C      READ IN THE ARRAYS WHICH DEFINE THE POINT TYPES.
93      C
94      DO 200 I=1,NX1
95      200  READ(5,103) (IU(I,J),J=1,NY)
96      C
97      103   FORMAT(50I1)
98      C
99      DO 201 I=1,NX
100     201  READ(5,103) (IV(I,J),J=1,NY1)
101     C
102     DO 202 I=1,NX
103     202  READ(5,103) (IS(I,J),J=1,NY)
104     C
105     DO 203 I=1,NX
106     203  READ(5,103) (IE(I,J),J=1,NY)
107     C
108     C
109     C      SET ALL WORKING ARRAYS TO ZERO.
110     C
111     DO 204 I=1,NX
112     DO 204 J=1,NY
113     C

```

```

114      DO 205 K=1,3
115      U(I,J,K)=0.
116      V(I,J,K)=0.
117      S(I,J,K)=0.
118      ETA(I,J,K)=0.
119      C1(I,J,K)=0.
120      H(I,J,K)=0.
121      ROW(I,J,K)=0.
122      205 CONTINUE
123      C
124      D(I,J)=0.
125      SIG(I,J)=0.
126      SIGC(I,J)=0.
127      UHAR(I,J)=0.
128      VBAR(I,J)=0.
129      SBAR(I,J)=0.
130      CBAR(I,J)=0.
131      EBAR(I,J)=0.
132      K1(I,J)=0.
133      K2(I,J)=0.
134      204 CONTINUE
135      C
136      NN=1
137      NSUM=0
138      C      READ IN THE DEPTHS AT S-POINTS (M.).
139      C
140      DO 206 I=1,NX
141      206 READ(5,104) (S(I,J),J=1,NY)
142      104 FORMAT(1UF5.0)
143      C
144      C      READ IN THE FRESH WATER SOURCES (M==3/S).
145      C
146      DO 208 I=1,NX
147      208 READ(5,104) (SIG(I,J),J=1,NY)
148      C
149      C
150      C      READ IN THE WIND SPEED AND DIRECTION (M/S AND DEGREES).
151      C      READ(5,105) WND / CURRENT CONVENTION
152      C      READ(5,105) DIRN
153      C
154      C      COMPUTE THE COMPONENTS OF WIND STRESS.
155      C
156      CC=1.3*10.**(-3.)
157      ROWA=1.2*10.**(-3.)
158      DIRN=(DIRN/180.)*3.14159
159      WX=CC*R0WA*10.*4.*WND*COS(DIRN)*WND
160      WY=CC*R0WA*10.*4.*WND*SIN(DIRN)*WND
161      DIRN=(DIRN/180.)/3.14159
162      C
163      C
164      C      READ IN THE HORIZONTAL EDDY STRESSES.
165      C
166      READ(5,106) N1,N2
167      106 FORMAT(2F10.0)
168      C
169      READ(5,101) IPL0T1
170      C

```

```

171 C      IF I PLOT 1 .EQ. 1 THE OUTPUT WILL BE PLOTTED.
172 C
173      READ(5,105) START
174 105  FORMAT(F5.0)
175 C
176 C
177 C      ESTABLISH THE INITIAL CONDITIONS.
178 C
179      CALL INIT
180 C
181 C      OUTPUT THE INITIAL VALUES.
182 C
183 C
184      WRITE(6,31)
185 31   FORMAT(1H1)
186 32   FORMAT(/)
187      WRITE(6,110)
188 110  FORMAT(5X,14H INITIAL VALUES//)
189      WRITE(6,111) DX
190 111  FORMAT(1UX,10H DELTAX = ,E10.3//)
191      WRITE(6,112) DY
192 112  FORMAT(1UX,10H DELTAY = ,E10.3//)
193      WRITE(6,113) DT
194 113  FORMAT(1UX,10H DELTAT = ,E10.3//)
195      WRITE(6,114) TOR
196 114  FORMAT(1UX,25H BOTTOM FRICTION. TOR = ,E10.3//)
197      WRITE(6,115) RRLAT
198 115  FORMAT(1UX,12H LATITUDE = ,F5.0//)
199      WRITE(6,116) WND,DJRN
200 116  FORMAT(1UX,14H WIND SPEED = ,F5.0,10X,18H WIND DIRECTION = )
201      WRITE(6,117) N1,N2
202 117  FORMAT(1UX,32H HORIZONTAL EDDY STRESSES N1 = }
203      $E10.3//,N2 = ,F5.0/1
204      WRITE(6,118) RK1,RK2
205 118  FORMAT(1UX,37H HORIZONTAL EDDY DIFFUSIVITIES K1 = ,
206      $E10.3,10X,6HK2 = ,E10.3//)
207      WRITE(6,31)
208 C
209 C      OUTPUT THE WATER DEPTHS.
210 C
211      WRITE(6,120)
212 120  FORMAT(1UX,21H WATER DEPTH IN METRES,/)
213 C
214      DO 215 I=1,NX
215      WRITE(6,32)
216 215  WRITE(6,119) (DC(I,J),J=1,NY)
217 119  FORMAT(1X,16F7.1)
218      WRITE(6,31)
219 C
220      WRITE(6,121)
221 121  FORMAT(1UX,28H FRESH WATER SOURCES (N**3/S) //)
222 C
223      DO 216 I=1,NX
224      WRITE(6,32)
225 216  WRITE(6,119) (SIG(I,J),J=1,NY)
226 C
227 C

```

```

228      C
229      C      CONVERT TO CGS UNITS.
230      C
231      C      DO 207 I=1,NX
232      C      DO 207 J=1,NY
233      C      D(I,J)=D(I,J)*10.**2.
234      C      H(I,J+1)=D(I,J)
235      C      H(I,J+2)=D(I,J)
236      C      SIG(I,J)=SIG(I,J)*10.**6.
237      207      CONTINUE
238      C
239      C
240      C
241      C
242      C
243      C
244      C      CALL OUTPUT
245      C
246      C
247      C      *** START THE MAIN COMPUTATIONAL LOOPS ***
248      C      ***
249      C      ***
250      C      ***
251      C      ***
252      C      ***
253      C
254      C      T0=T0R
255      C      TMAX=UU.J
256      C
257      C      DO 300 NN=1,NSTEP
258      C
259      C      T=FL0AT(NN)*DT
260      C      TI=T/3600.
261      C      TC=TI/12.4224
262      C
263      C
264      C      TOR=T0+TMAX/EXP(TC)
265      C
266      C
267      C
268      C      CALL BC
269      C
270      C      CALL ETA3
271      C
272      C
273      C      CALL UVEL
274      C
275      C      CALL VVEL
276      C
277      C      CALL PARAM
278      C
279      C
280      C      *** ALL VARIABLES HAVE NOW BEEN UPDATED ***
281      C
282      C      UPDATE THE TIME INDEX.
283      C
284      C      DO 720 I=1,NX

```

```

285      DO 720 J=1,NY
286      U(I,J+1)=U(I,J+2)
287      U(I,J+2)=U(I,J+3)
288      V(I,J+1)=V(I,J+2)
289      V(I,J+2)=V(I,J+3)
290      ETA(I,J,1)=ETA(I,J+2)
291      ETA(I,J,2)=ETA(I,J+3)
292      H(I,J,1)=H(I,J+2)
293      H(I,J,2)=H(I,J+3)
294      720 CONTINUE
295      C
296      C
297      C TEST FOR OUTPUT.
298      C
299      C IF (TC .LE. START) GO TO 300
300      C
301      C IF (MOD(NN,NANS) .NE. 0) GO TO 699
302      C
303      C PRINT OUT RESULTS.
304      C
305      C CALL OUTPUT
306      699 CONTINUE
307      C
308      C
309      300 CONTINUE
310      C
311      C IF (IPLOT1 .EQ. 0) GO TO 1255
312      C CALL PLOT(0.,0.,999)
313      1255 CONTINUE
314      C
315      C
316      STOP
317      END

```

```

1      SUBROUTINE INIT
2
3      C
4      C SET THE INITIAL CONDITIONS.
5
6      C
7      C
8      C INSERT THE INITIAL VALUES.
9
10     C
11     C
12     C
13     C
14     C
15     C
16     C

```

DO 100 I=1,NX
 DO 100 J=1,NY
 DO 100 K=1,3
 IF (IS(I,J) .EQ. 1) GO TO 100
 ROW(I,J,K)=1.
 100 CONTINUE
 C
 RETURN
 END

```

1      SUBROUTINE OUTPUT
2
3      C
4      C      OUTPUT THE RESULTS.
5      C
6      WRITE(6,31)
7      31  FORMAT(1H1)
8      32  FORMAT(/)
9      C
10     C      OUTPUT THE TIME IN HOURS.
11     C
12     WRITE(6,25) TIM, TC
13     25  FORMAT(1OX+25HELAPSED TIME IN HOURS = ,F6.2+50X+
14     & 26 NUMBER OF TIDAL CYCLES = ,F6.2/)

15     C
16     C
17     WRITE(6,100)
18     100 FORMAT(5X+18HSURFACE ELEVATIONS/)
19     DO 150 I=1,NX
20     WRITE(6,32)
21     150 WRITE(6,101) (ETA(I,J+2),J=1,NY)
22     101 FORMAT(1X+16F7.1)
23     C
24     C
25     WRITE(6,31)
26     WRITE(6,102)
27     102 FORMAT(1OX+31HU-COMPONENT HORIZONTAL VELOCITY,///)
28     C
29     DO 151 I=1,NX1
30     WRITE(6,32)
31     151 WRITE(6,101) (U(I,J+2),J=1,NY)
32     C
33     WRITE(6,31)
34     WRITE(6,103)
35     103 FORMAT(1OX+31HV-COMPONENT HORIZONTAL VELOCITY,///)
36     C
37     DO 152 I=1,NX
38     WRITE(6,32)
39     152 WRITE(6,101) (V(I,J+2),J=1,NY1)
40     C
41     C
42     C+++
43     WRITE(6,31)
44     WRITE(6,104)
45     104 FORMAT(1OX+24HSTRATIFICATION PARAMETER,///)
46     C
47     DO 153 I=1,NX
48     WRITE(6,32)
49     153 WRITE(6,101) (S(I,J+2),J=1,NY)
50     C
51     C      ++++++ ++++++ ++++++ ++++++ ++++++ ++++++
52     C      ++++++ ++++++ ++++++ ++++++ ++++++ ++++++
53     C
54     IF (IPILOT1 .EQ. 0) GO TO 125
55     CALL GRAPH
56     CONTINUE
125

```

57 C
58 C ++++++
59 C ++++++
60 C
61 RETURN
62 END

1 SUBROUTINE BC
2 C
3 C SPECIFY THE TIDAL AMPLITUDES AND
4 C PHASES AROUND THE OPEN BOUNDARY.
5 C
6 C
7 C SCALE=0.60
8 C
9 C ET A(1, 6, 3)=200.*SCALE*SIN(0.5058*T IM -0.0175*240.)
10 C
11 C
12 C DO V=SIN(0.5058*T IM)
13 C
14 C
15 C ET A(9, 12, 3)=250.*SCALE*D0V
16 C ET A(10, 12, 3)=300.*SCALE*D0V
17 C ET A(11, 12, 3)=350.*SCALE*D0V
18 C
19 C
20 C A=120.
21 C TT=0.5058*TIM
22 C
23 C ET A(8, 1, 3)=A*SCALE*SIN(TT-0.0175*150.)
24 C ET A(9, 1, 3)=A*SCALE*SIN(TT-0.0175*140.)
25 C ET A(10, 1, 3)=A*SCALE*SIN(TT-0.0175*130.)
26 C ET A(11, 1, 3)=A*SCALE*SIN(TT-0.0175*125.)
27 C ET A(12, 1, 3)=A*SCALE*SIN(TT-0.0175*120.)
28 C ET A(13, 1, 3)=A*SCALE*SIN(TT-0.0175*120.)
29 C ET A(14, 1, 3)=A*SCALE*SIN(TT-0.0175*115.)
30 C ET A(15, 1, 3)=A*SCALE*SIN(TT-0.0175*110.)
31 C ET A(16, 1, 3)=A*SCALE*SIN(TT-0.0175*105.)
32 C ET A(17, 1, 3)=A*SCALE*SIN(TT-0.0175*100.)
33 C ET A(17, 2, 3)=A*SCALE*SIN(TT-0.0175*100.)
34 C ET A(17, 3, 3)=A*SCALE*SIN(TT-0.0175*100.)
35 C ET A(17, 4, 3)=A*SCALE*SIN(TT-0.0175*100.)
36 C ET A(17, 5, 3)=A*SCALE*SIN(TT-0.0175*100.)
37 C ET A(17, 6, 3)=A*SCALE*SIN(TT-0.0175*100.)
38 C ET A(17, 7, 3)=A*SCALE*SIN(TT-0.0175*100.)
39 C ET A(17, 8, 3)=A*SCALE*SIN(TT-0.0175*100.)
40 C
41 C
42 C
RE TURN
END

```

1      SUBROUTINE ETA3
2
3      C
4      C      UPDATE THE SURFACE ELEVATIONS.
5      C
6      DO 100 I=1,NX
7      DO 100 J=1,NY
8      C
9      IF(IE(I,J) .EQ. 1) GO TO 100
10     IF(IE(I,J) .EQ. 2) GO TO 101
11     C
12     C      COMPUTATIONAL POINT.
13     C
14     C1=S IG(I,J)/(4.*DX*DY*ROW(I,J+2))
15     C2=((H(I,J+2)+H(I+1,J+2))*U(I,J+2)-(H(I-1,J+2)+H(I,J+2))
16     S*U(I-1,J+2))/(4.*DX)
17     C3=((H(I,J+2)+H(I+J+1,J+2))*V(I,J+2)-(H(I+J-1,J+2)+H(I,J+2))
18     S*V(I,J-1,J+2))/(4.*DY)
19     C
20     C
21     C
22     C      ETA(I,J+3)=ETA(I,J+1)+2.*DT*(C1+C2+C3)
23     C
24     101  CONTINUE
25     H(I,J+3)=ETA(I,J+3)+D(I,J)
26     C
27     100  CONTINUE
28     RETURN
29     END

```

```

1      SUBROUTINE UVEL
2      C
3      C      UPDATE THE U COMPONENTS.
4      C
5      REAL MB1,MB2,MB3,MB4,MB5,MB6,MB7
6      DO 100 I=1,NX1
7      DO 100 J=1,NY
8      C
9      IF (IU(I,J) .EQ. 1) GO TO 100
10     C
11     C      OUTSIDE THE GRID OR ON A BOUNDARY.
12     C
13     MB3=WX
14     C
15     MB6=-G*(H(I,J,2)+H(I+1,J,2))*(ETA(I+1,J,2)-ETA(I,J,2))/(4.*DX)
16     MB7=-G*(H(I,J,2)+H(I+1,J,2))*2.0*(ROW(I+1,J,2)-ROW(I,J,2))
17     S*(ROW(I,J,2)+ROW(I+1,J,2))**(-1.0)/(8.*DX)
18     C
19     C      NEAR AN OPEN E-W BOUNDARY OR A CORNER.
20     C
21     IF (IU(I,J) .EQ. 3 .OR. IU(I,J) .EQ. 4) GO TO 20
22     C
23     C      NORMAL EQUATION
24     MB1=N1*(H(I+1,J+1)*(U(I+1,J+1)-U(I,J+1))-H(I,J+1)*(U(I,J+1)
25     S-U(I-1,J+1)))/(4.*DX**2.0)
26     GO TO 21
27     C
28     20     MB1=U.
29     21     CONTINUE
30     C
31     C
32     C      NEAR AN OPEN N-S BOUNDARY OR A CORNER.
33     C
34     IF (IU(I,J) .EQ. 2 .OR. IU(I,J) .EQ. 4) GO TO 22
35     C
36     C      NORMAL POINT.
37     HDSH1=0.25*(H(I,J+1)+H(I+1,J+1)+H(I+1,J+1,1)+H(I+1,J+1,1))
38     HDSH2=0.25*(H(I,J-1,1)+H(I,J-1)+H(I+1,J-1)+H(I+1,J-1,1))
39     MB2=N2*(HDSH1*(U(I,J+1)-U(I,J+1))-HDSH2*(U(I,J+1)-U(I,J-1)))
40     S)/(4.*DY**2.0)
41     C
42     GO TO 23
43     C
44     22     MB2=0.
45     23     CONTINUE
46     C
47     C
48     C
49     C      NEAR AN OPEN N-S BOUNDARY OR A CORNER.
50     C
51     IF (IU(I,J) .EQ. 2 .OR. IU(I,J) .EQ. 4) GO TO 24
52     C
53     U1=U(I,J+1)
54     U2=0.25*(V(I,J-1,1)+V(I,J-1)+V(I+1,J-1)+V(I+1,J-1,1))
55     MB4=-0.5*(ROW(I,J+1)+ROW(I+1,J+1))+TOR*U1*SORT(U1**2.+U2**2.)
56     GO TO 25

```

```

57      C
58      24      MB 4=-U .5*(ROW(I,J,1)+ROW(I+1,J+1))*TOR*
59      S(U(I,J,1)*ABS(U(I+J+1))
60      25      CONTINUE
61      C
62      C
63      C      NEAR AN OPEN N-S GUY OR A CORNER.
64      C
65      IF (IU(I,J) .EQ. 2 .OR. IU(I,J) .EQ. 4) GO TO 26
66      C
67      MB 5=+F*(H(I,J,2)+H(I+1,J,2))*(V(I,J,2)+V(I+1,J,2)
68      S+V(I+1,J-1,2)+V(I,J-1,2))/8.
69      C
70      GO TO 27
71      26      MB 5=U.
72      C
73      27      CONTINUE
74      C
75      C
76      U(I,J,3)=(0.5*(H(I,J+1)+H(I+1,J+1))+U(I,J+1)
77      S+2.*DT*(MB1+MB2+MB3+MB4+MB5+MB6+MB7))/((0.5*(H(I,J,3)
78      S+H(I+1,J,3)))
79      C
80      C
81      100     CONTINUE
82      RETURN
83      END

```

```

1      SUBROUTINE VVEL
2 C
3 C      UPDATE THE V COMPONENTS.
4 C
5      REAL MB1,MB2,MB3,MB4,MB5,MB6,MB7
6      DO 100 I=1,NX
7      DO 100 J=1,NY1
8 C
9 C      OUTSIDE THE GRID OR ON A BOY.
10 C
11      IF (IV(I,J) .EQ. 1) GO TO 100
12 C
13      MB3=WY
14 C
15      MB6=-G*(H(I+J+1,2)+H(I+J,2))*(ETA(I,J+1,2)-ETA(I,J,2))/(4.*DY)
16 C
17      MB7=-G*(H(I,J,2)+H(I,J+1,2))*2.*((ROW(I,J+1,2)-ROW(I,J,2))
18      $/(ROW(I,J,2)+ROW(I,J+1,2))/(8.*DY)
19 C
20 C
21 C      NEAR AN OPEN E-W BOY OR A CORNER.
22 C
23      IF (IV(I,J) .EQ. 3 .OR. IV(I,J) .EQ. 4) GO TO 20
24      HDSH1=u.25*(H(I,J,1)+H(I+J+1,1)+H(I+1,J+1,1)+H(I+1,J,1))
25      HDSH2=0.25*(H(I-1,J,1)+H(I-1,J+1,1)+H(I,J+1,1)+H(I,J,1))
26      MB1=N1*(HDSH1*(V(I+1,J,1)-V(I,J+1))-HDSH2*(V(I,J,1)-V(I-1,J,1)))
27      $/(4.*DX*2.)
28 C
29      GO TO 21
30 C
31      20      MB1=0.
32 C
33      21      CONTINUE
34 C
35 C      NEAR AN OPEN N-S BOY OR A CORNER.
36 C
37      IF (IV(I,J) .EQ. 2 .OR. IV(I,J) .EQ. 4) GO TO 22
38 C
39      MB2=N2*(H(I,J+1,1)*(V(I,J+1,1)-V(I,J,1))-H(I,J,1)*
40      $(V(I,J+1)-V(I,J-1,1)))/(4.*DY*2.)
41 C
42      GO TO 23
43 C
44      22      MB2=0.
45 C
46      23      CONTINUE
47 C
48 C      NEAR AN OPEN E-W BOY OR A CORNER.
49 C
50      IF (IV(I,J) .EQ. 3 .OR. IV(I,J) .EQ. 4) GO TO 24
51 C
52      U1=U.25*(U(I-1,J,1)+U(I-1,J+1,1)+U(I,J+1,1)+U(I,J,1))
53      U2=V(I,J,1)
54 C
55      MB4=-0.5*(ROW(I,J,1)+ROW(I,J+1,1))+TOR*U2*SQRT(U1**2.+U2**2.)
56      GO TO 25

```

```

57 C
58 24 MB4=-0.5*(ROW(I,J+1)+ROW(I+J+1,1))+TOR=V(I,J+1)*ABSC(V(I,J+1))
59 25 CONTINUE
60 C
61 C
62 C NEAR AN OPEN E-W BAY OR A CORNER.
63 C
64 IF (IV(I,J) .EQ. 3 .OR. IV(I,J) .EQ. 4) GO TO 26
65 C
66 MB5=-F*(H(I,J,2)+H(I,J+1,2))+U(I-1,J,2)+U(I-1,J+1,2)
67 +U(I,J+1,2)+U(I,J,2))/8.
68 C
69 GO TO 27
70 26 MB5=0.
71 C
72 27 CONTINUE
73 C
74 C
75 C
76 V(I,J,3)=(V(I,J,1)+(H(I,J,1)+H(I,J+1,1))/2.+2.*DT*
77 +(MB1+MB2+MB3+MB4+MB5+MB6+MB7))/((H(I,J,3)+H(I,J+1,3))/2.)
78 C
79 C
80 100 CONTINUE
81 C
82 RETURN
83 END

```

```

1 SUBROUTINE PARAM
2 C
3 C COMPUTE THE STRATIFICATION PARAMETER.
4 C
5 C
6 DO 100 I=2,NX1
7 DO 100 J=2,NY1
8 C
9 IF (IS(I,J) .EQ. 1) GO TO 100
10 C
11 C(I,J,2)=(0.5*(U(I-1,J,2)+U(I,J,2))+2.
12 +0.5*(V(I,J-1,2)+V(I,J,2)))*2.)
13 C
14 C(I,J,2)=SQRT(C(I,J,2))
15 C
16 STRAT=C(I,J,2)**3./D(I,J)+0.001
17 C(I,J,2)= ALOG10(STRAT)
18 C
19 IF (C(I,J,2).GT.S(I,J,2)) S(I,J,2)=C(I,J,2)
20 C
21 100 CONTINUE
22 C
23 RETURN
24 END

```

```

1      SUBROUTINE GRAPH
2 C
3 C      DO THE PLOTTING ON A CALCOMP*960.
4 C
5 C
6 C
7 C      DL EV      THE CONTOUR SPACING
8 C      HG TC      THE HEIGHT OF THE CONTOUR LABEL IN INCHES.
9 C      INDC       PERMITS CONTOUR LABELS TO VARY DURING A SINGLE PLOT.
10 C     LABC .GE. 0  NO. OF DIGITS IN CONTOUR LABEL DECIMAL PART.
11 C             = -1  NO DECIMAL PART.
12 C             = -3  NO LABEL PRINTED ON THE CONTOUR LINE.
13 C     LWGT=1      CONTOUR DRAWN BY ORDINARY LINE.
14 C             =2    HEAVY LINE.
15 C             =3    DASHED LINE.
16 C     NARC=1      VARIES 1-10 CAN BE USED TO SMOOTH THE CONTOURS.
17 C     NLEV        SUPPLIED BY GETLEV. IT IS THE NO. OF CONTOURS DRAWN.
18 C     NX,NY       WORKING DIMENSIONS OF THE ARRAY BEING CONTOURED.
19 C     X1PL,Y1PL   COORDINATES OF THE Z ORIGIN, I.E Z(1,1).
20 C     XLPL,YLPL   COORDINATES OF Z(NX,NY).
21 C     Z           ARRAY TO BE CONTOURED.
22 C     ZLEV        ARRAY SUPPLIED BY GETLEV.
23 C
24 C
25 C      THE FOLLOWING ARE THE PLOT OPTIONS.
26 C
27 C     IPLOT1=0    NO PLOT.
28 C     IPLOT1=1    PLOT ELEVATIONS AND VELOCITY VECTORS ONLY.
29 C     IPLOT1=2    ALSO PLOT SALINITY.
30 C
31 C
32 C
33     DD X=0.7947
34     DD Y=0.9959
35 C
36     FC X=2.*UDX*FLOAT(NX1)
37     FC Y=2.*UDY*FLOAT(NY1)
38     XSHFT=2.*UDX*FLOAT(NX)+2.
39     YSHFT=2.*UDY*FLOAT(NY)+2.
40 C
41 C      SET THE PLOTTING PARAMETERS.
42 C
43     INDC=1
44     LWGT=1
45     HG TC=0.4
46     LABC=-1
47     NARC=1
48 C
49 C
50 C      IF NN=0 THEN CONTOUR THE BOTTOM TOPOGRAPHY.
51 C
52     IF (NN .EQ. 0) GO TO 100
53 C
54     GO TO 101
55 C
56 150  CONTINUE

```

```

57 C
58 C      CONTOUR THE BOTTOM DEPTHS.
59 C
60      CALL NEWPLT(4, "SWA", 3, 0)
61      CALL FACTOR(0.5)
62 C
63      DO 102 I=1,NX
64      DO 102 J=1,NY
65      Z(I,J)=D(I,J)/10.**2.
66      IF (IS(I,J) .EQ. 1) Z(I,J)=10.**35.
67 102    CONTINUE
68 C
69 C      SET CONTOUR SPACING AT 500 M.
70      OLEV=500.
71 C
72 C
73 C
74      CALL GETLEV(Z,NX,NY,OLEV,ZLEV,NLEV)
75 C
76 C      CONTOUR THE DEPTH.
77 C
78      CALL PLOT(4.,7.,-3)
79      CALL CONTR(Z,NX,NY,0.,0.,FCX,FCY,ZLEV,LABC,LWGT,NLEV,HGTC,
80      $NARC)
81 C
82 C      DRAW THE BOUNDARY.
83 C
84      CALL BOUND
85 C
86      CALL SYMBOL(0.,-2.,0.28,16HDX      DY      DT,0.,+16)
87      CALL SYMBOL(5.0,-2.,0.28,23HTOK      WX      WY      F,
88      <0.,+23)
89 C
90      PD X=DX*10.**(-5.)
91      PD Y=DY*10.**(-5.)
92 C
93      CALL NUMBER(0.,-3.,0.28,PDX,0.,+2)
94      CALL NUMBER(2.,-3.,0.28,PDY,0.,+2)
95      CALL NUMBER(4.,-3.,0.28,DT,0.,+2)
96      CALL NUMBER(6.,-3.,0.28,TOK,0.,+4)
97      CALL NUMBER(8.,-3.,0.28,WX,0.,+2)
98      CALL NUMBER(10.,-3.,0.28,WY,0.,+2)
99      CALL NUMBER(12.,-3.,0.28,F,0.,+6)
100 C
101     CALL PLOT(0.,0.,999)
102 C
103 101    CONTINUE
104 C
105     IF (IPLOT1 .EQ. 1) CALL NEWPLT(4,"SWA",3,0)
106     IF (IPLOT1 .EQ. 2 .OR. IPLOT1 .EQ. 3)
107     $CALL NEWPLT(3,"SWA",3,0)
108     CALL FACTOR(0.5)
109 C
110     CALL PLOT(4.,7.,-3)
111 C
112 C      CONTOUR THE SURFACE ELEVATIONS.
113 C

```

```

114      DO 110 I=1,NX
115      DO 110 J=1,NY
116      Z(I,J)=ETA(I,J,2)
117      IF (I.EQ.I) .EQ. 1) Z(I,J)=10.*35.
118 110  CONTINUE
119 C
120 C      SET THE CONTOUR SPACING AT 100 CM.
121 C
122 DLEV=100.
123 C
124 CALL GETLEV(Z,NX,NY,DLEV,ZLEV,NLEV)
125 C
126 CALL CONTR(Z,NX,NY,O.,0.,FCX,FCY,ZLEV,LABC,LWGT,NLEV,HGTC,
127 $NARC)
128 C
129 C
130 C      DRAW THE VELOCITY VECTORS.
131 C
132 USCALE=0.02
133 C
134 DO 120 I=1,NX1
135 DO 120 J=1,NY1
136 C
137 UMOD=U(I,J,2)**2.+U(I,J+1,2)**2.+V(I,J,2)**2.+V(I+1,J,2)**2.
138 C
139 IF (UMOD .EQ. 0.) GO TO 120
140 C
141 C
142 XCD=DO X*2.*FLOAT(I-1)*DDX
143 YCD=DO Y*2.*FLOAT(J-1)*DDY
144 C
145 CALL SYMBOL(XCD,YCD,0.16,4,0.,-1)
146 C
147 UBAR=0.5*(U(I,J,2)+U(I,J+1,2))
148 VBAR=0.5*(V(I,J,2)+V(I+1,J,2))
149 C
150 C
151 XDSH=XCD+UBAR*USCALE
152 YDSH=YCD+VBAR*USCALE
153 C
154 CALL PLOT(XDSH,YDSH,2)
155 C
156 120 CONTINUE
157 C
158 C
159 C      PLOT THE BOUNDARY.
160 C
161 CALL BOUND
162 C
163 CALL SYMBOL(1.,-2.,0.28,4HTC,0.,+4)
164 CALL NUMBER(1.,-3.,0.28,TC,0.,+2)
165 CALL SYMBOL(3.,-2.,0.28,5HTM,0.,+5)
166 CALL NUMBER(3.,-3.,0.28,TIM,0.,+2)
167 C
168 C      CONTOUR THE STRATIFICATION PARAMETER.
169 C
170 IF ((IPLOT1 .EQ. 2) .OR. (IPLOT1 .EQ. 3)) GO TO 130

```

171 GO TO 131
172 130 CONTINUE
173 C
174 CALL PLOT(0.,YSHFT - 3)
175 C
176 DO 132 I=1,NX
177 DO 132 J=1,NY
178 Z(I,J)=S(I+J+2)
179 132 CONTINUE
180 C
181 C SET CONTOUR SPACING AT 1
182 C
183 DLEV=1.0
184 C
185 CALL GETLEV(Z,NX,NY,DLEV,ZLEV,NLEV)
186 C
187 C
188 CALL CONTR(Z,NX,NY,0.,0.,FCX,FCY,ZLEV,LABC,LWGT,NLEV,HGTC,
189 \$MARC)
190 C
191 C
192 131 CONTINUE
193 C
194 C DRAW THE BOUNDARY.
195 C
196 IF ((IPL0T1 .EQ. 2) .OR. (IPL0T1 .EQ. 3)) GO TO 150
197 GO TO 151
198 150 CONTINUE
199 C
200 CALL BOUND
201 C
202 151 CONTINUE
203 C
204 CALL PLOT(0.,0.,999)
205 C
206 C
207 RETURN
208 END

```

1      SUBROUTINE BOUND
2 C
3 C      DRAWS THE COASTAL BOUNDARY.
4 C
5      COMMON/XC10/XA(20),YA(20),XB(20),YB(20),XC(20),YC(20),
6      XD(20),YE(20),XE(20),YE(20),XF(20),YF(20)
7      DA TA(XA(J),J=1,7)/0.,2.4,2.4,8.7,8.7,0.,0.,1./
8      DA TA(YA(J),J=1,7)/3.,3.,1.,1.,0.,0.,1./
9      DA TA(XB(J),J=1,9)/0.,2.4,2.4,8.7,8.7,10.3,10.3,0.,1./
10     DA TA(YB(J),J=1,9)/9.,9.,7.,7.,5.,5.,0.,0.,1./
11     DA TA(XC(J),J=1,20)/0.,0.8,0.8,2.4,2.4,5.5,5.5,7.1,
12     S 7.,1.,8.,7.,8.,7.,10.,3.,1.,0.,3.,11.,9.,11.,9.,13.,4.,13.,4.,15.,0.,0.,1.,
13     DA TA(YC(J),J=1,20)/11.,11.,11.,13.,13.,15.,15.,11.,11.,13.,13.,
14     S 9.,9.,13.,13.,13.,11.,11.,9.,9.,0.,0.,1.,
15     DA TA(XD(J),J=1,15)/0.,0.8,0.8,2.4,2.4,7.1,7.1,11.,9.,11.,9.,
16     S 13.,4.,13.,4.,15.,0.,15.,0.,0.,1.,
17     DA TA(YD(J),J=1,15)/17.,17.,17.,19.,19.,21.,21.,
18     S 23.,23.,23.,21.,21.,13.,13.,9.,9.,0.,0.,1.,
19     DA TA(XE(J),J=1,15)/13.,4.,13.,4.,16.,6.,16.,6.,18.,2.,18.,2.,
20     S 16.,6.,16.,6.,19.,8.,19.,8.,22.,9.,22.,9.,25.,3.,0.,0.,1.,
21     DA TA(YE(J),J=1,15)/23.,9.,23.,23.,23.,21.,21.,21.,19.,
22     S 19.,17.,17.,11.,11.,15.,15.,0.,0.,1.,
23 C
24 C
25 C      DRAW THE BOUNDARY OF THE GRID.
26 C
27     CALL PLOT(0.,0.,3)
28     CALL PLOT(25.3,0.,2)
29     CALL PLOT(25.3,0,23.9,2)
30     CALL PLOT(0.,23.9,0,2)
31     CALL PLOT(0.,0.,2)
32 C
33 C
34     CALL LINE(XA,YA,5,1,0,0)
35     CALL LINE(XB,YB,7,1,0,0)
36     CALL LINE(XC,YC,18,1,0,0)
37     CALL LINE(XD,YD,23,1,0,0)
38     CALL LINE(XE,YE,13,1,0,0)
39     RETURN
40     END

```

1 27.78
2 33.34
3 74.5344
4 0017
5 0013
6 002200
7 0300
8 11 11 13 11 11 11 1
9 11 11 15 51 11 11 1
10 11 11 55 55 11 11 1
11 11 11 55 11 11 11 1
12 11 11 55 11 11 11 1
13 11 11 51 11 11 11 1
14 11 15 51 11 11 11 1
15 25 55 55 11 11 11 1
16 25 55 51 11 11 12 1
17 25 55 51 15 55 52 1
18 25 55 55 55 51 51 1
19 25 55 55 55 51 11 1
20 25 55 55 11 11 11 1
21 25 55 55 11 11 11 1
22 25 55 55 11 11 11 1
23 43 33 33 33 11 11 1
24 11 11 11 11 11 11
25 11 11 15 11 11 11
26 11 11 55 51 11 11
27 11 11 55 51 11 11
28 11 11 51 11 11 11
29 11 11 55 11 11 11
30 11 15 11 11 11 11
31 25 55 55 11 11 11
32 25 55 51 11 11 11
33 25 55 11 15 55 51
34 25 55 55 55 55 51
35 25 55 55 55 11 11
36 25 55 55 55 11 11
37 25 55 51 11 11 11
38 25 55 51 11 11 11
39 25 55 55 51 11 11
40 43 33 33 31 11 11
41 11 11 12 11 11 11 1
42 11 11 13 31 11 11 1
43 11 11 33 33 11 11 1
44 11 11 33 33 11 11 1
45 11 11 33 11 11 11 1
46 11 11 33 31 11 11 1
47 11 13 31 11 11 11 1
48 23 33 33 31 11 11 1
49 23 33 33 11 11 12 1
50 23 33 31 13 33 32 1
51 23 33 33 33 33 32 1
52 23 33 33 33 31 31 1
53 23 33 33 33 31 11 1
54 23 33 33 11 11 11 1
55 23 33 33 11 11 11 1
56 23 33 33 33 11 11 1

57	2222222211111
58	1111121111111
59	1111133111111
60	1111333311111
61	1111333311111
62	1111331111111
63	1111333111111
64	1113311111111
65	2333333111111
66	2333331111121
67	2333311333321
68	2333333333321
69	2333333331311
70	23333333311111
71	2333331111111
72	2333331111111
73	23333333111111
74	22222222111111
75	.
76	.
77	.
78	.
79	.
80	.
81	.
82	.
83	.
84	.
85	.
86	.
87	.
88	.
89	36. 18. 73. 82. 100. 45. 18.
90	.
91	137. 117. 95. 95. 92. 60.
92	9.
93	146. 91. 73. 110. 82.
94	45. 11.
95	120. 104. 110. 110. 44.
96	36. 7.
97	128. 146. 120. 128. 110.
98	92. 84. 55. 18.
99	146. 160. 120. 146. 120. 106.
100	88. 18. 9.
101	920. 160. 150. 110. 120. 92.
102	.
103	2470. 1830. 360. 126. 106. 92.
104	.
105	4350. 3300. 1850. 360. 137. 130.
106	92. 11.
107	4400. 4200. 4400. 4600. 2600. 2500.
108	150. 92.
109	.
110	.
111	.
112	.
113	.

114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143 00.00
144 90.00
145 10 00 00 .00 10 00 00 .00
146 0002
147 03.20